ECL 4340

POWER SYSTEMS

LECTURE 21

OPTIMAL POWER FLOW, LINEAR PROGRAMMING

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ANNOUNCEMENTS

- Be reading Chapter 6, sections 6.12 & 6.13.
- HW 11 is uploaded, due November 30.
- Final is on Monday, December 12, 9:00 - 11:00 a.m. in Room 204

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OPTIMAL POWER FLOW (OPF)

- Next we'll consider OPF and SCOPF
- OPF functionally combines the power flow with economic dispatch
- SCOPF adds in contingency analysis
- In both minimize a cost function, such as operating cost, taking into account realistic equality and inequality constraints
- Equality constraints
  - bus real and reactive power balance
  - generator voltage setpoints
  - area MW interchange

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OPF, cont'd

- Inequality constraints
  - transmission line/transformer/interface flow limits
  - generator MW limits
  - generator reactive power capability curves
  - bus voltage magnitudes (not yet implemented in Simulator OPF)
- Available Controls
  - generator MW outputs
  - transformer taps and phase angles
  - reactive power controls

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Two Example OPF Solution Methods

- Non-linear approach using Newton's method
  - handles marginal losses well, but is relatively slow and has problems determining binding constraints
  - Generation costs (and other costs) represented by quadratic or cubic functions
- Linear Programming
  - fast and efficient in determining binding constraints, but can have difficulty with marginal losses.
  - used in PowerWorld Simulator
  - Generation costs (and other costs) represented by piecewise linear functions

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LP OPF Solution Method

- Solution iterates between
  - solving a full ac power flow solution
    - enforces real/reactive power balance at each bus
    - enforces generator reactive limits
    - system controls are assumed fixed
    - takes into account non-linearities
  - solving a primal LP
    - changes system controls to enforce linearized constraints while minimizing cost

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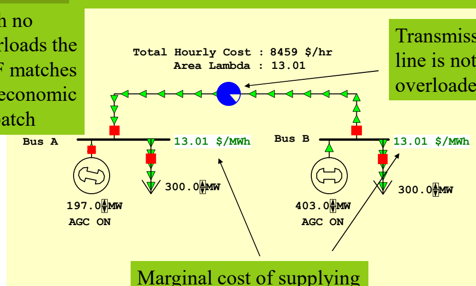
Two Bus with Unconstrained Line

With no overloads the OPF matches the economic dispatch

Transmission line is not overloaded



Marginal cost of supplying power to each bus (locational marginal costs)

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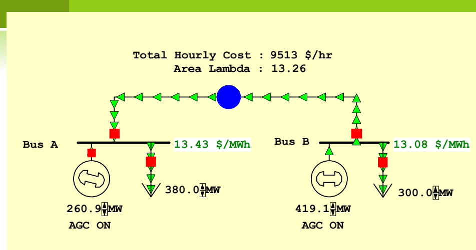
Two Bus with Constrained Line



With the line loaded to its limit, additional load at Bus A must be supplied locally, causing the marginal costs to diverge.

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Three Bus (B3) Example

- Consider a three bus case (bus 1 is system slack), with all buses connected through 0.1 pu reactance lines, each with a 100 MVA limit
- Let the generator marginal costs be
  - Bus 1: 10 $ / MWhr; Range = 0 to 400 MW
  - Bus 2: 12 $ / MWhr; Range = 0 to 400 MW
  - Bus 3: 20 $ / MWhr; Range = 0 to 400 MW
- Assume a single 180 MW load at bus 2

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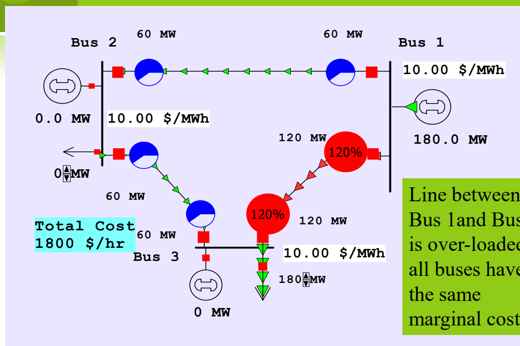
B3 with Line Limits NOT Enforced



Line between Bus 1and Bus 3 is over-loaded; all buses have the same marginal cost

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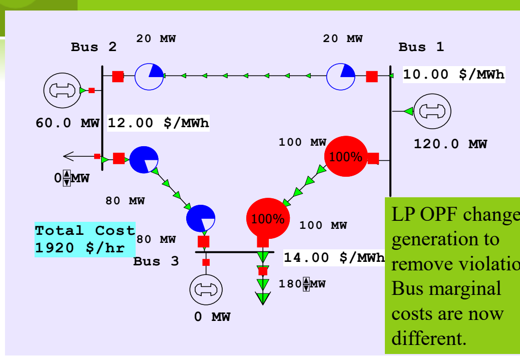
B3 with Line Limits Enforced



LP OPF changes generation to remove violation. Bus marginal costs are now different.

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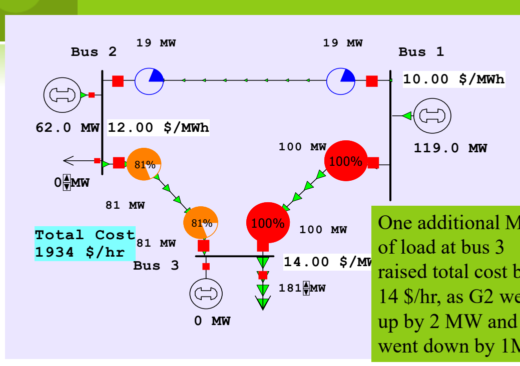
Verify Bus 3 Marginal Cost



One additional MW of load at bus 3 raised total cost by 14 $/hr, as G2 went up by 2 MW and G1 went down by 1MW

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Why is bus 3 LMP = $14 /MWh

- All lines have equal impedance. Power flow in a simple network distributes inversely to impedance of path.
  - For bus 1 to supply 1 MW to bus 3, 2/3 MW would take direct path from 1 to 3, while 1/3 MW would “loop around” from 1 to 2 to 3.
  - Likewise, for bus 2 to supply 1 MW to bus 3, 2/3MW would go from 2 to 3, while 1/3 MW would go from 2 to 1to 3.

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Why is bus 3 LMP $ 14 / MWh, cont’d

- With the line from 1 to 3 limited, no additional power flows are allowed on it.
- To supply 1 more MW to bus 3 we need
  - Pg1 + Pg2 = 1 MW
  - 2/3 Pg1 + 1/3 Pg2 = 0; (no more flow on 1-3)
- Solving requires we up Pg2 by 2 MW and drop Pg1 by 1 MW -- a net increase of $14.

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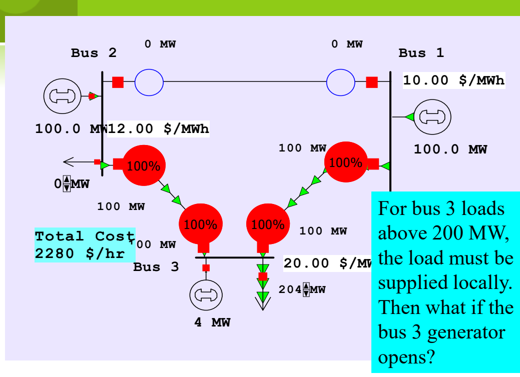
Both lines into Bus 3 Congested



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Quick Coverage of Linear Programming

- LP is probably the most widely used mathematical programming technique
- It is used to solve linear, constrained minimization (or maximization) problems in which the objective function and the constraints can be written as linear functions

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Example Problem 1

- Assume that you operate a lumber mill which makes both construction-grade and finish-grade boards from the logs it receives.
- Suppose it takes 2 hours to rough-saw and 3 hours to plane each 1000 board feet of construction-grade boards. Finish-grade boards take 2 hours to rough-saw and 5 hours to plane for each 1000 board feet.
- Assume that the saw is available 8 hours per day, while the plane is available 15 hours per day. If the profit per 1000 board feet is \$100 for construction-grade and \$120 for finish-grade, how many board feet of each should you make per day to maximize your profit?

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Problem 1 Setup

Let x_1=amount of cg, x_2= amount of fg

Maximize $100x_1 + 120x_2$

s.t. $2x_1 + 2x_2 \le 8$

$3x_1 + 5x_2 \le 15$

$x_1, x_2 \ge 0$

Notice that all of the equations are linear, but they are inequality, as opposed to equality, constraints; we are seeking to determine the values of x_1 and x_2

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Example Problem 2

- A nutritionist is planning a meal with 2 foods: A and B.
- Each ounce of A costs $ 0.20, and has 2 units of fat, 1 of carbohydrate, and 4 of protein.
- Each ounce of B costs $0.25, and has 3 units of fat, 3 of carbohydrate, and 3 of protein.
- Provide the least cost meal which has no more than 20 units of fat, but with at least 12 units of carbohydrates and 24 units of protein.

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Problem 2 Setup

Let x_1=ounces of A, x_2= ounces of B

Minimize $0.20x_1 + 0.25x_2$

s.t. $2x_1 + 3x_2 \le 20$

$x_1 + 3x_2 \ge 15$

$4x_1 + 3x_2 \ge 24$

Again all of the equations are linear, but they are inequality, as opposed to equality, constraints; we are again seeking to determine the values of x_1 and x_2; notice there are also more constraints then solution variables

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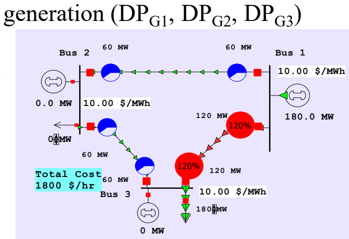
Three Bus Case Formulation

- For the earlier three bus system given the initial condition of an overloaded transmission line, minimize the cost of generation (DP_{G1}, DP_{G2}, DP_{G3}) such that the change in generation is zero, and the flow on the line between buses 1 and 3 is not violating its limit



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Three Bus Case Problem Setup

Let $x_1 = \Delta P_{G1}$, $x_2 = \Delta P_{G2}$, $x_3 = \Delta P_{G3}$

Minimize $10x_1 + 12x_2 + 20x_3$

s.t. $\frac{2}{3}x_1 + \frac{1}{3}x_2 \le -20$ Line flow constraint

$x_1 + x_2 + x_3 = 0$ Power balance constraint

enforcing limits on x_1, x_2, x_3

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LP Standard Form

The standard form of the LP problem is

Minimize $\mathbf{c}\,\mathbf{x}$

s.t. $\mathbf{A}\mathbf{x} = \mathbf{b}$

$\mathbf{x} \ge \mathbf{0}$

Maximum problems can be treated as minimizing the negative

where $\mathbf{x}$ = n-dimensional column vector

$\mathbf{c}$ = n-dimensional row vector

$\mathbf{b}$ = m-dimensional column vector

$\mathbf{A}$ = m×n matrix

For the LP problem usually $n \gg m$

The previous examples were not in this form!

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Replacing Inequality Constraints with Equality Constraints

- The LP standard form does not allow inequality constraints
- Inequality constraints can be replaced with equality constraints through the introduction of slack variables, each of which must be greater than or equal to zero

$$\ldots \le b_i \rightarrow \ldots + y_i = b_i \quad \text{with } y_i \ge 0$$
$$\ldots \ge b_i \rightarrow \ldots - y_i = b_i \quad \text{with } y_i \ge 0$$

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Lumber Mill Example with Slack Variables

- Let the slack variables be x_3 and x_4, so

Minimize $-(100x_1 + 120x_2)$ Minimize the negative

s.t. $2x_1 + 2x_2 + x_3 = 8$

$3x_1 + 5x_2 + x_4 = 15$

$x_1, x_2, x_3, x_4 \geq 0$

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LP Definitions

A vector $\mathbf{x}$ is said to be basic if

1. $\mathbf{Ax} = \mathbf{b}$
2. At most m components of $\mathbf{x}$ are non-zero

if there as less than m non-zeros the $\mathbf{x}$ is called degenerate

Define $\mathbf{x} = \begin{bmatrix} \mathbf{x}_\text{B} \\ \mathbf{x}_\text{N} \end{bmatrix}$ (with $\mathbf{x}_\text{B}$ basic) and $\mathbf{A} = [\mathbf{A}_B \quad \mathbf{A}_N]$

$\mathbf{A}_\text{B}$ is called the basis matrix

With $[\mathbf{A}_B \quad \mathbf{A}_N]\begin{bmatrix} \mathbf{x}_\text{B} \\ \mathbf{x}_\text{N} \end{bmatrix} = \mathbf{b}$ so $\mathbf{x}_\text{B} = \mathbf{A}_B^{-1}(\mathbf{b} - \mathbf{A}_N\mathbf{x}_\text{N})$

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LP Definitions

A vector $\mathbf{x}$ is said to be feasible if

1. $\mathbf{Ax} = \mathbf{b}$
2. $\mathbf{x} \geq \mathbf{0}$

A basic solution is not necessarily feasible, and a feasible solution is not necessarily basic.

We'll also assume $\mathbf{A}$ is full rank, so $\mathbf{Ax} = \mathbf{b}$ always has a solution

An m by n matrix (with m < n) is rank m if there are m linearly independent columns

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Fundamental LP Theorem

- Given an LP in standard form with **A** of rank m then
  - If there is a feasible solution, there is a basic feasible solution
  - If there is an optimal, feasible solution, then there is an optimal, basic feasible solution
- Note, there could be a LARGE number of basic, feasible solutions
  - Simplex algorithm determines the optimal, basic feasible solution

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Simplex Algorithm

- The key is to move intelligently from one basic feasible solution to another, with the goal of continually decreasing the cost function
- The algorithm does this by determining the "best" variable to bring into the basis; this requires that another variable exit the basis, while retaining a basic, feasible solution

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At the Solution

- At the solution we have m non-zero elements of x, $\mathbf{x}_B$, and n-m zero elements of $\mathbf{x}$, $\mathbf{x}_N$
- We also have $\mathbf{x}_B = \mathbf{A}_B^{-1}(\mathbf{b} - \mathbf{A}_N\mathbf{x}_N)$
- And we define $\boldsymbol{\lambda} = \mathbf{c}_B\mathbf{A}_B^{-1}$
- The elements of $\boldsymbol{\lambda}$ tell the incremental cost of enforcing each constraint (just like in economic dispatch)

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Lumber Mill Example Solution

Minimize $-(100x_1 + 120x_2)$

s.t. $2x_1 + 2x_2 + x_3 = 8$

$3x_1 + 5x_2 + x_4 = 15$

$x_1, x_2, x_3, x_4 \geq 0$

An initial basic feasible solution is $x_1 = 0, x_2 = 0, x_3 = 8, x_4 = 15$

The solution is $x_1 = 2.5, x_2 = 1.5, x_3 = 0, x_4 = 0$

Then $\boldsymbol{\lambda} = [100 \quad 120]\begin{bmatrix} 2 & 2 \\ 3 & 5 \end{bmatrix}^{-1} = \begin{bmatrix} 35 \\ 10 \end{bmatrix}$

Economic interpretation of λ is the profit is increased by 35 for every hour we up the first constraint (the saw) and by 10 for every hour we up the second constraint (plane)

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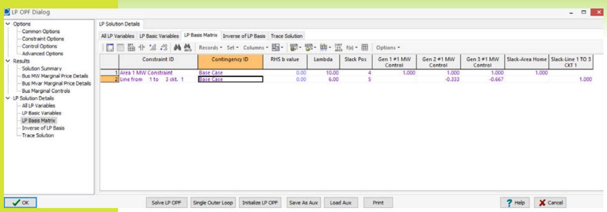
LP Sensitivity Matrix (A Matrix) for Three Bus Example



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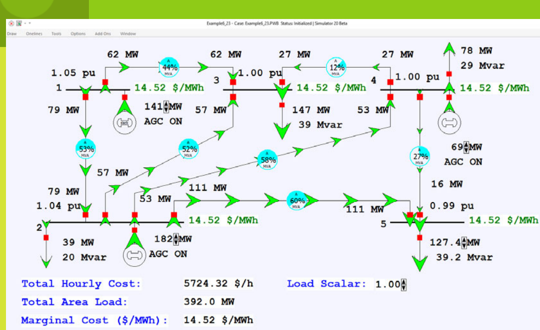
Example 6_23 Optimal Power Flow



In the example the load is gradually increased

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Locational Marginal Costs (LMPs)

- In an OPF solution, the bus LMPs tell the marginal cost of supplying electricity to that bus
- The term "congestion" is used to indicate when there are elements (such as transmission lines or transformers) that are at their limits; that is, the constraint is binding
- Without losses and without congestion, all the LMPs would be the same
- Congestion or losses causes unequal LMPs
- LMPs are often shown using color contours; a challenge is to select the right color range!

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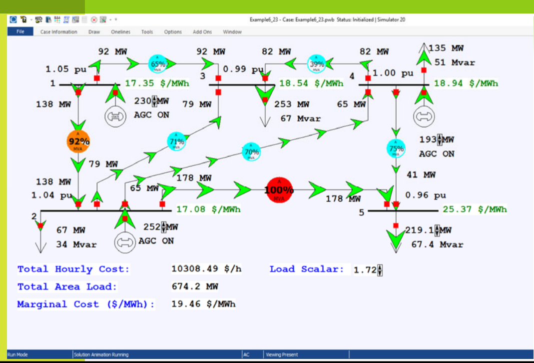
Example 6_23 Optimal Power Flow with Load Scale = 1.72



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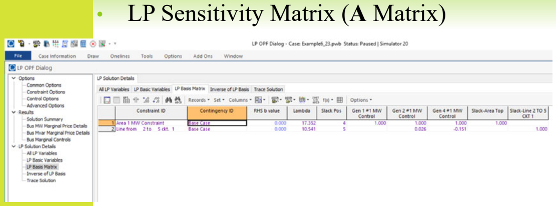
Example 6_23 Optimal Power Flow with Load Scale = 1.72

- LP Sensitivity Matrix (**A** Matrix)



The first row is the power balance constraint, while the second row is the line flow constraint. The matrix only has the line flows that are being enforced.

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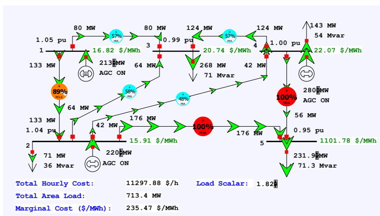
Example 6_23 Optimal Power Flow with Load Scale = 1.82

- This situation is infeasible, at least with available controls. There is a solution because the OPF is allowing one of the constraints to violate (at high cost)

						Control	Control	Control		CKT 1
1	Area 1 MW Constraint	Base Case	0.000	16.824	4	1.000	1.000	1.000	1.000	
2	Line from 2 to 5 ckt. 1	Base Case	0.000	993.664	5		0.026	-0.146		1.000
3	Line from 4 to 5 ckt. 1	Base Case	-0.002	1000.000	6		-0.024	0.140		



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